

Corrections to mass scale predictions in SO(10) GUT with higher dimensional operators

Alakabha Datta^{a)}, Sandip Pakvasa^{a)} and Utpal Sarkar^{b)}

^{a)} *Physics Department, University of Hawaii at Manoa, 2505 Correa Road, Honolulu, HI 96822, USA.*

^{b)} *Theory Group, Physical Research Laboratory, Ahmedabad - 380009, India.*

Abstract

We calculate the two loop contributions to the predictions of the mass scales in an SO(10) grand unified theory. We consider the modified unification scale boundary conditions due to the non-renormalizable higher dimensional terms arising from quantum gravity or spontaneous compactification of extra dimensions in Kaluza-Klein type theory. We find the range of these couplings which allows left-right symmetry to survive till very low energy (as low as $\sim \text{TeV}$) and still be compatible with the latest values of $\sin^2 \theta_W$ and α_s derived from LEP. We consider both the situation when the left-right parity is broken and conserved. We consider both supersymmetric and non-supersymmetric versions of the SO(10) theory. Taking the D-conserved non-susy case as an example we calculate the effects of moderate threshold uncertainties at the heavy scale, due to the unknown higgs masses, on the gravity induced couplings.

There are many extensions of the standard model, which are suggested on various aesthetical grounds. But so far experiments could not find anything which is not predicted by the standard model. In other words, the standard model is consistent with all the experiments carried out so far, although there are appealing reasons to believe that there is physics beyond the standard model. In the standard model the (V-A) nature of the theory is put in by hand, whereas in a left-right symmetric extension [1] of the standard model this comes about through spontaneous symmetry breaking.

In the left-right symmetric extension of the standard model, at higher energies the gauge group is extended to a left-right symmetric group $G_{LR} \equiv SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. When appropriate higgs fields acquire vacuum expectation value (vev), this group breaks down to one of its subgroup $G_{std} \equiv SU(2)_L \otimes U(1)_Y$. There will then be new scalar and gauge particles of mass of the order of this symmetry breaking scale M_R . The mixing of these gauge bosons with the standard model gauge bosons puts lower bound on this scale.

The $K_L - K_S$ mass difference gives a lower bound [2] of about 1.6 TeV on M_R from the box diagram with both W_L and W_R exchanges. However, this constraint is subject to the assumption of manifest left-right symmetry, which is to assume that the Kobayashi-Maskawa matrices of the left- and the right- handed sectors are same. In absence of this artificial symmetry (which does not have any natural explanation) the bound [3] on M_R is relaxed to 300 GeV. From the direct search [4] at CDF the lower bound on M_{W_R} is 520 GeV. This bound is not applicable to left-right symmetric models where the W_R couples only to the heavy neutrinos, which again decay very fast. The strongest bound on M_R comes from an analysis [5, 6] of the precision measurement of the Z -pole from the CERN e^+e^- collider LEP [7]. From a fit of the 1992 data and for the commonly chosen higgs triplet fields for the left-right symmetry breaking, the lower bound on M_R is of the order of TeV.

In the standard model the three gauge coupling constants are free parameters and are

all different. This has a natural explanation in grand unified theories [8] in which the strong and the electroweak interaction are only low energy manifestations of a single interaction. The GUT interaction is a gauge interaction based on a simple gauge group with only one gauge coupling constant. Through spontaneous symmetry breaking this breaks down to a low energy symmetry group. Then the different coupling constants evolve in different ways to give the present day low energy coupling constants. Some of the attractive features of GUTS were their natural explanation of the problem of baryogenesis, and their unique prediction of proton decay. However, proton decay has not yet been observed and the question of baryogenesis took a completely different shape following the observation of large anomalous baryon number nonconservation at high temperatures in the presence of sphaleron fields. The main interest in GUTs remains is its unification of coupling constants and charge quantization.

Recently there has again been an upsurge of interest [9, 10, 11, 12, 13] in GUTs following the precision measurement of the three gauge coupling constants at LEP. The normalised gauge coupling constants for the groups $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, as obtained [7] from analyzing the LEP data, are given by,

$$\begin{aligned}\alpha_1(M_z) &= .16887 \pm .000040 \\ \alpha_2(M_z) &= .03322 \pm .00025 \\ \alpha_3(M_z) &= .120 \pm .007\end{aligned}\tag{1}$$

respectively. With the minimal particle content, it is not possible to unify all the three coupling constants at any energy. This apparently rules out [9] minimal $SU(5)$ GUT and any GUTs without any intermediate scales and new particles unless the effect of gravity modifies the situation.

It was further pointed out that the scale of the intermediate symmetry breaking can be severely constrained by the present values of the gauge coupling constants. For the minimal supersymmetric GUTs, the supersymmetry breaking scale $M_R \sim 1$ TeV gives a good fit [9] to evolve all the gauge couplings to an unification point. However, threshold effects and higher order corrections make this scale uncertain by orders of magnitude. This makes the threshold effects and higher order corrections very important in studying the evolution of the gauge coupling constants in the light of the LEP data.

It was pointed out that if one studies any GUTs with left-right symmetric group G_{LR} as one of its intermediate symmetry group, then the present LEP data severely constrain [10] this symmetry breaking scale M_R . For any GUTs and any number of new symmetries above M_R , one obtains a lower bound

$$M_R > 10^9 \text{ GeV}.$$

This bound can be relaxed [14] if one breaks the left-right parity and the left-right symmetric group G_{LR} at different scales.

If now signatures of the right handed gauge bosons are found in the next generation accelerators (since the experimental lower bound is only around a TeV), that will not however mean that there is inconsistency in GUTs. It was shown that in a very specific supersymmetric $SO(10)$ GUT one can satisfy [13] the unification constraint with low M_R . The details of this deserves further study.

Since the GUT scale is very close to the Planck scale, the effects of gravity may not be negligible. It was shown that if effects of gravity are considered through higher dimensional operators, then even the minimal $SU(5)$ GUT with no new particle content may be consistent [16] with the LEP data and proton decay.

We have studied [12] the effect of gravity to see if the constraints on M_R can be relaxed. We considered higher dimensional nonrenormalizable operators which may arise due

to quantum gravity or spontaneous compactification of extra dimensions in the Kaluza-Klein type theory and their effect in the $SO(10)$ lagrangian. The GUT scale boundary condition was found to be modified and for certain choice of parameters low M_R could be made consistent with $SO(10)$ GUT. In this paper we present details of our analysis. Here we include the threshold effects and study the two loop evolution of the coupling constants which are also very significant in these analyses. First we present the formalism and then present our analysis. At the end we summarize our results.

Higher dimensional operators were considered originally [15] to help solve some problems in fermion masses. The idea is to find out if the low energy physics contains some signatures of gravity effects. In all these analysis the coupling constants in these nonrenormalizable terms are free parameters. Someday we may learn if such coupling constants may arise from gravity naturally.

In our analysis we consider dimension five and dimension six operators when the contribution from dimension five operator vanish. We note that the effect of all operators higher than dimension six can be absorbed in the couplings of the dimension six operators and hence their inclusion does not increase the number of parameters. We therefore consider only dimension five and dimension six operators in our analysis.

The main objective of our study is to look for consistency of low M_R . For this purpose we consider the symmetry breaking chain,

$$\begin{aligned}
SO(10) &\xrightarrow{M_U} SU(4) \times SU(2)_L \times SU(2)_R \\
&[\equiv G_{PS}] \\
&\xrightarrow{M_I} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}
\end{aligned}$$

$$[\equiv G_{LR}]$$

$$\begin{aligned} \xrightarrow{M_R} \quad & SU(3)_c \times SU(2)_L \times U(1)_Y \\ & [\equiv G_{std}] \end{aligned}$$

$$\xrightarrow{M_W} \quad SU(3)_c \times U(1)_{em}. \quad (2)$$

Near the scale $M_U \sim 10^{16}$ GeV or higher the gravity effects are not negligible. But we assume that any theory beyond this scale respects the $SO(10)$ symmetry. Then the lagrangian will contain all the usual $SO(10)$ invariant dimension 4 interaction terms and in addition will contain $SO(10)$ invariant higher dimensional nonrenormalizable terms. These higher dimensional terms will be suppressed by the Planck scale (in theories [15] where these terms are induced by quantum gravity) or by the Kaluza–Klein compactification scale (in theories [17] where these terms are induced by spontaneous compactification of the extra dimensions in the Kaluza-Klein type theories), which can even be two orders of magnitude below the Planck scale.

The lagrangian can be written as,

$$L = L_R + L_{NR} \quad (3)$$

where the first part of the lagrangian contains all the renormalizable dimension 4 terms including the $SO(10)$ gauge invariant term,

$$L = -\frac{1}{2}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) \quad (4)$$

where,

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \\ A_\mu &= A_\mu^i \frac{\lambda_i}{2} \end{aligned}$$

with

$$\text{Tr}(\lambda_i \lambda_j) = \frac{1}{2} \delta_{ij},$$

where, the λ 's are the $SO(10)$ generators. The nonrenormalizable part of the lagrangian contains all the higher dimensional $SO(10)$ invariant terms. We are presently interested in only dimension 5 and 6 terms, which are given as,

$$L = \sum_{n=5} L^{(n)}$$

$$L^{(5)} = -\frac{1}{2} \frac{\eta^{(1)}}{M_{Pl}} \text{Tr}(F_{\mu\nu} \phi F^{\mu\nu}) \quad (5)$$

$$L^{(6)} = -\frac{1}{2} \frac{1}{M_{Pl}^2} \left[\eta_a^{(2)} \{ \text{Tr}(F_{\mu\nu} \phi^2 F^{\mu\nu}) + \text{Tr}(F_{\mu\nu} \phi F^{\mu\nu} \phi) \} + \right. \\ \left. \eta_b^{(2)} \text{Tr}(\phi^2) \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \eta_c^{(2)} \text{Tr}(F^{\mu\nu} \phi) \text{Tr}(F_{\mu\nu} \phi) \right] \quad (6)$$

where $\eta^{(n)}$ are dimensional couplings of the higher dimensional operators. When any higgs scalar ϕ acquires *vev* ϕ_0 , these operators induce effective dimension 4 terms, which modifies the boundary conditions at the scale ϕ_0 .

Let us consider the symmetry breaking chain[2] at the scale M_U . We shall first consider the case when this symmetry breaking is mediated by the *vev* of a 54-plet of higgs. In this case the left-right parity is broken at M_R only when $SU(2)_R$ is broken and the gauge coupling constants g_L and g_R corresponding to the groups $SU(2)_L$ and $SU(2)_R$ respectively evolve similarly between M_U and M_R so that $g_L(M_R) = g_R(M_R)$. In the second case we shall consider the symmetry breaking at the scale M_U by a 210-plet of higgs. This breaks the discrete left-right parity symmetry [14] D, so that g_L and g_R evolve in a different way below M_U and as a result one obtains $g_L(M_R) \neq g_R(M_R)$.

In the D-conserving case, the symmetry breaking at M_U takes place when the 54-plet higgs Σ of $SO(10)$ acquires a *vev*,

$$\langle \Sigma \rangle = \frac{1}{\sqrt{30}} \Sigma_0 \text{diag}(1, 1, 1, 1, 1, 1, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}). \quad (7)$$

where, $\Sigma_0 = \sqrt{\frac{6}{5\pi\alpha_G}} M_U$ and $\alpha_G = g_0^2/4\pi$ is the GUT coupling constant. We now introduce the parameters,

$$\epsilon^{(1)} = \left[\left\{ \frac{1}{25\pi\alpha_G} \right\}^{\frac{1}{2}} \frac{M_U}{M_{Pl}} \right] \eta^{(1)} \quad (8)$$

and

$$\epsilon_i^{(2)} = \left[\left\{ \frac{1}{25\pi\alpha_G} \right\}^{\frac{1}{2}} \frac{M_U}{M_{Pl}} \right]^2 \eta_i^{(2)} \quad (9)$$

Then the G_{PS} invariant effective lagrangian will be modified by these higher dimensional operators as follows,

$$\begin{aligned} & -\frac{1}{2}(1 + \epsilon_4) \text{Tr}(F_{\mu\nu}^{(4)} F^{(4)\mu\nu}) - \frac{1}{2}(1 + \epsilon_2) \text{Tr}(F_{\mu\nu}^{(2L)} F^{(2L)\mu\nu}) \\ & - \frac{1}{2}(1 + \epsilon_2) \text{Tr}(F_{\mu\nu}^{(2R)} F^{(2R)\mu\nu}) \end{aligned} \quad (10)$$

where,

$$\epsilon_4 = \epsilon^{(1)} + \epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}$$

and

$$\epsilon_2 = -\frac{3}{2}\epsilon^{(1)} + \frac{9}{4}\epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}.$$

Then the usual G_{PS} lagrangian can be recovered with the modified coupling constants,

$$\begin{aligned} g_4^2(M_U) &= \bar{g}_4^2(M_U)(1 + \epsilon_4)^{-1} \\ g_{2L}^2(M_U) &= \bar{g}_{2L}^2(M_U)(1 + \epsilon_2)^{-1} \\ g_{2R}^2(M_U) &= \bar{g}_{2R}^2(M_U)(1 + \epsilon_2)^{-1} \end{aligned} \quad (11)$$

where, \bar{g}_i are the coupling constants in the absence of the nonrenormalizable terms and g_i are the physical coupling constants that evolve below M_U . Similarly, the physical gauge fields are defined as, $A'_i = A_i\sqrt{1 + \epsilon_i}$.

The *vev* of Σ leaves unbroken a larger symmetry group than G_{PS} , which is $O(6) \otimes O(4)$. The D-parity is thus unbroken and hence $SU(2)_L$ and $SU(2)_R$ always receive equal

contributions. Furthermore, since overall contributions to all the gauge groups cannot change the predictions of $\sin^2 \theta_w$ and α_s , the *vev* of Σ can only contribute to one combination of the couplings, *i.e.* the relative couplings of $SU(4)$ and the $SU(2)$ s. For this reason no matter how many higher dimensional terms we consider, what contributes to the low energy predictions of $\sin^2 \theta_w$ and α_s is only the combination,

$$\epsilon = \epsilon_4 - \epsilon_2. \quad (12)$$

If we now assume that the dimension 6 terms $\epsilon_i^{(2)}$ are negligible compared to the dimension 5 terms $\epsilon^{(1)}$, then we further get,

$$\epsilon_4 = \frac{2}{5}\epsilon \quad \text{and} \quad \epsilon_2 = -\frac{3}{5}\epsilon.$$

As we argued earlier, this does not reduce the number of parameters in the theory. If we include the higher dimensional terms, then the allowed region in ϵ will be shared by the other $\epsilon^{(n)}$ s.

It was pointed out in ref. [12] that for any choice of the parameter ϵ it was not possible to have a consistent theory with low M_R . It was necessary to make the symmetry breaking scale M_I very close to M_U , so that higher dimensional operators can introduce another parameter, which can then allow low M_R .

The *vev* of a 45-plet field H can break the symmetry group G_{PS} to G_{LR} ,

$$\langle H \rangle = \frac{1}{\sqrt{12}i} H_0 \begin{pmatrix} 0_{33} & 1_{33} & 0_{34} \\ -1_{33} & 0_{33} & 0_{34} \\ 0_{43} & 0_{43} & 0_{44} \end{pmatrix} \quad (13)$$

where, 0_{mn} is a $m \times n$ null matrix and 1_{mm} is a $m \times m$ unit matrix. The antisymmetry of the matrix H will imply that to dimension five operators there is no contribution from this higgs. The lowest order contribution comes from the dimension six operators,(In Ref 12. the dimension five operator was taken to give the lowest order contribution. This is

incorrect, since due to the antisymmetry of the 45 representation the dimension five operator is zero.)

$$L'^{(2)} = -\frac{1}{2} \frac{1}{M_{Pl}^2} \left[\eta_a'^{(2)} \text{Tr}(F_{\mu\nu} H^2 F^{\mu\nu}) + \eta_b'^{(2)} \text{Tr}(H^2) \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \eta_c'^{(2)} \text{Tr}(F^{\mu\nu} H) \text{Tr}(F_{\mu\nu} H) \right] \quad (14)$$

The vev of H doesnot modify the $SU(2)$ couplings. The $SU(4)$ invariant effective lagrangian will only contain a new contribution,

$$L''^{(2)} = -\frac{1}{2} \frac{1}{M_{Pl}^2} \left[\eta_a'^{(2)} \text{Tr}(F_{\mu\nu} \phi_{15}^2 F^{\mu\nu}) + \eta_b'^{(2)} \text{Tr}(\phi_{15}^2) \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \eta_c'^{(2)} \text{Tr}(F^{\mu\nu} \phi_{15}) \text{Tr}(F_{\mu\nu} \phi_{15}) \right] \quad (15)$$

where, ϕ_{15} transforms as $(15, 1, 1)$ of G_{PS} . At M_I the symmetry group $SU(4)_c$ breaks down to $SU(3)_c \otimes U(1)_{B-L}$ when the field ϕ_{15} acquires a vev ,

$$\phi_{15} = \frac{1}{\sqrt{24}} \phi_0 \text{diag}[1, 1, 1, -3]. \quad (16)$$

with, $\phi_0 = \sqrt{6/5\pi\alpha_4} M_I$. We now define,

$$\epsilon_i'^{(2)} = \frac{\eta_i'^{(2)} \phi_0^2}{24 M_{Pl}^2} = \left[\frac{1}{20\pi\alpha_4} \left[\frac{M_I}{M_{Pl}} \right]^2 \right] \eta_i'^{(2)} \quad (17)$$

where, $i = a, b, c$. The $SU(3)_c \otimes U(1)_{B-L}$ invariant kinetic energy term for the gauge bosons will then be given by,

$$-\frac{1}{2} (1 + \epsilon'_3) \text{Tr}(F_{\mu\nu}^{(3)} F^{(3)\mu\nu}) - \frac{1}{2} (1 + \epsilon'_1) \text{Tr}(F_{\mu\nu}^{(1)} F^{(1)\mu\nu}) \quad (18)$$

where,

$$\epsilon'_3 = \epsilon_a'^{(2)} + 12\epsilon_b'^{(2)}$$

and

$$\epsilon'_1 = 7\epsilon_a'^{(2)} + 12\epsilon_b'^{(2)} + 12\epsilon_c'^{(2)}.$$

In general, ϵ'_1 and ϵ'_3 may be treated as two free parameters. But we shall assume $\epsilon_b'^{(2)} = \epsilon_a'^{(2)} = \epsilon_c'^{(2)}$ and hence,

$$\epsilon'_3 = 0.42 \quad \epsilon'_1 = \epsilon'(\text{say}). \quad (19)$$

Thus the parameter space in ϵ' and ϵ we present here may be further relaxed to some extent. However, the number of parameters in the $\sin^2 \theta_w$ and α_s is not changed and we cannot expect any change in low energy predictions. In our analysis we shall present the parameter space of ϵ' and ϵ , which allows low M_R . For the D-nonconserved case a 210 plet of higgs is used to break $SO(10)$ to the group G_{PS} without D-parity conservation. The vev of the Higgs is given by,

$$\langle H_{210} \rangle = \frac{1}{\sqrt{32}} H_0 \text{diag}(1_{44}, 1_{44}, -1_{44}, -1_{44}). \quad (20)$$

where H_0 is related to the vector boson mass M_X by $\sqrt{\frac{2}{\pi\alpha_G}} M_X = H_0$. Keeping only the dim-5 operator we get,

$$\epsilon_4 = 0$$

$$\epsilon_{2L} = -\epsilon_{2R} = 8\epsilon^{(1)} = \epsilon$$

where,

$$\epsilon^{(1)} = \sqrt{\frac{2}{32\pi\alpha_G}} \left[\frac{M_X}{M_{Pl}} \right]$$

For the evolution of the coupling constants we use the two loop renormalization group equations [9, 11, 18],

$$\mu \frac{\partial}{\partial \mu} \alpha_i(\mu) = \frac{2}{4\pi} \left[b_i + \sum_j \frac{b_{ij}}{4\pi} \alpha_j(\mu) \right] \alpha_i^2(\mu) + \frac{2b_{ij}}{(4\pi)^2} \alpha_i^3 \quad (21)$$

where i, j index represents the different subgroups at the energy scale μ and $\alpha_i = \frac{1}{4\pi} g_i^2$. The various β -functions with SUSY and without SUSY are given in Ref 19 [19]. We use the survival hypothesis[20] to find the Higgs content at the various mass scales for any given chain. In table 1. we show the higgs bosons that live at different mass scales.

Table 1: Higgs spectrum at various mass scales for the D-conserved and the D-nonconseved chain

<i>Group G_i</i>	<i>Higgs content</i>
$(2_L 2_R 4_C P)$	$(2, 2, 1)_{10}$ $(1, 3, \bar{10})_{126}$ $(3, 1, 10)_{126}$ $(1, 1, 15)_{45}$
$(2_L 2_R 4_C)$	$(2, 2, 1)_{10}$ $(1, 3, \bar{10})_{126}$ $(1, 1, 15)_{45}$
$(2_L 2_R 1_{B-L} 3_C P)$	$(2, 2, 0, 1)_{10}$ $(1, 3, 2, 1)_{126}$ $((3, 1, 2, 1)_{126})$
$(2_L 2_R 1_{B-L} 3_C)$	$(2, 2, 0, 1)_{10}$ $(1, 3, 2, 1)_{126}$

An approximate solution of the evolution equation can be written as,

$$\begin{aligned}
\alpha_i^{-1}(\mu') &= \beta_0 \ln \frac{\mu'}{\mu} + \frac{1}{\alpha_i(\mu)} + \frac{\beta_1}{\beta_0} \ln \left[\frac{\alpha^{-1}(\mu') + \frac{\beta_1}{\beta_0}}{\alpha^{-1}(\mu) + \frac{\beta_1}{\beta_0}} \right] \\
\beta_0 &= \frac{1}{2\pi} \left[b_i + \sum b_{ij} \alpha_j(\mu) \right] \\
\beta_1 &= -\frac{2b_{ii}}{(4\pi)^2}
\end{aligned} \tag{22}$$

At each symmetry breaking threshold we use the following matching conditions for the couplings when the group G breaks to the group G_i [21]

$$\alpha_i^{-1}(\mu) = \alpha_G^{-1} - \frac{\lambda_i}{12\pi} \tag{23}$$

where,

$$\lambda_i = C_G - C_{G_i} + \text{Tr}(\theta_i^H)^2 \ln \frac{M_H}{\mu}$$

θ_i^H are the generators of G_i for the representation in which the higgs, M_H , appear. C_G and C_{G_i} are the quadratic Casimir invariants for the group G and the group G_i while μ is

Table 2: The Higgs bosons at M_U

$SO(10)$ multiplet	$G_{2,2,4}$ multiplet
54	$S_1(1, 1, 1), S_2(1, 1, 20'), S_3(3, 3, 1)$
45	$\phi_1(3, 11), \phi_2(2, 2, 6), \phi_3(1, 3, 1)$
126	$\Sigma_1(2, 2, 15), \Sigma_2(1, 1, 6)$

Table 3: The Higgs bosons at M_I

$SO(10)$ multiplet	$G_{2_L, 2_R, 1_{B-L}, 3_C}$ multiplet
45	$Y_1(1, 1, 1, 0), Y_2(1, 1, 0, 8), \bar{X}(1, 1, \frac{4}{3}, \bar{3}), X(1, 1, -\frac{4}{3}, 3)$
126	$\xi_1(3, 1, -\frac{2}{3}, 6), \xi_2(1, 3, \frac{2}{3}, \bar{6}), \xi_3(3, 1, \frac{2}{3}, 3), \xi_4(1, 3, -\frac{2}{3}, \bar{3})$

the symmetry breaking scale. Gravity induced corrections change $\alpha_i^{-1}(\mu)$ to $\alpha_i^{-1}(\mu)(1 + \epsilon_i)^{-1}$ as in eqn.(11). In our analysis we identify μ , the unification scale with the vector boson mass. Threshold corrections will occur due to the non-degeneracy of the higgs masses with the vector boson mass. Using the D-conserved non-susy case as an example we have calculated the effect of threshold corrections at the heavy scales M_I and M_U . The higgs masses are assumed to vary between $\frac{1}{5}$ and 5 times the vector boson mass. The threshold corrections enter through the factors λ_i appearing in eqn.24. The higgs that live around the mass scale M_U and M_I are given in table 3. and table 4.

Defining $\eta_H = \ln \frac{M_H}{M_X}$ one can write at M_U ,

$$\begin{aligned}
\lambda_4 &= (4 + 16\eta_{S_2} + 8\eta_{\phi_2} + 32\eta_{\Sigma_1} + 2\eta_{\Sigma_2} + 2\eta_{H'}) \\
\lambda_2 &= (6 + 12\eta_{S_3} + 12\eta_{\phi_2} + 30\eta_{\Sigma_1} + 4\eta_{\phi_1})
\end{aligned} \tag{24}$$

At the scale M_I one has,

$$\begin{aligned}
\lambda_3 &= (1 + 6\eta_{Y_2} + 15\eta_{\xi_1} + 15\eta_{\xi_2} + 3\eta_{\xi_3} + 3\eta_{\xi_4}) \\
\lambda_{B-L} &= (4 + 6\eta_{\xi_1} + 6\eta_{\xi_2} + 3\eta_{\xi_3} + 3\eta_{\xi_4})
\end{aligned}$$

$$\begin{aligned}
\lambda_{2L} &= (24\eta_{\xi_1} + 12\eta_{\xi_3}) \\
\lambda_{2R} &= (24\eta_{\xi_2} + 12\eta_{\xi_4})
\end{aligned} \tag{25}$$

The quantities $\lambda_4 - \lambda_2$ and $\lambda_3 - \lambda_{B-L}$ appear in the solution for M_U and M_I respectively. We consider two cases where the higgs masses are chosen such that the above quantities are at their extreme values. We further make the assumption that the higgs at a given scale coming from the same $SO(10)$ multiplet have the same masses. In the first case we choose $M_\Sigma, M_S, M_{H'}$ to be $(\frac{1}{5})M_U$ while M_ϕ to be $5M_U$. At M_I we choose M_Y and M_ξ to be $\frac{1}{5}M_I$. For the second case we just flip the higgs bosons around at the two scales. We refer to these two cases as case(a) and case(b). We have only considered the cases when M_I is not equal to M_U .

0.1 Results

Using the values of the standard model couplings at M_Z (eqn.1), the evolution equations and the matching conditions (eqn.24 and eqn.25) we find regions in the ϵ, ϵ' space which allow a low M_R for various values of the intermediate scale M_I and the unification scale M_U . In fig 1. the allowed regions for D-conserved non-susy case are shown. For M_U not equal to M_I the effects of threshold corrections have been included. In fig 2. and fig 3. the allowed regions for D-conserved and D-non-conserved susy case are shown. For the D-broken non-susy case the width of the allowed regions are too small to be shown graphically and therefore we present the results for this case in table 4. For the supersymmetric version the allowed regions are larger but no solution was found for the case $M_I = 10^{16}, M_U = 10^{18}$. Even though we have not carried out a full analysis of the threshold effects, from the examples considered, we do not expect moderate threshold effects to alter the regions in the parameter space drastically. In conclusion we have shown that both for the D-conserved and D-nonconserved case we can find regions in the parameter space of gravity induced couplings that allow M_R in the TeV range.

Table 4: Allowed ranges for ϵ and ϵ' for D-broken non-susy case

$\epsilon' (10^{-3})$	$\epsilon (10^{-3})$	M_I	M_U	$\frac{\alpha_{2L}(M_R)}{\alpha_{2R}(M_R)}$
4.92 – 10.38	-10 – -5.43	10^{16}	10^{17}	1.4
11.69 – 12.46	-9.72 – -1.032	10^{16}	10^{18}	1.3
6.46 – 8.46	-7.09 – -5.32	10^{17}	10^{17}	1.3
11.08 – 13.23	-9.59 – -7.91	10^{17}	10^{18}	1.2
9.23 – 10.92	-9.18 – -7.75	10^{17}	10^{18}	1.3
8.23 – 10.4	-7.33 – -5.50	10^{18}	10^{18}	1.2

0.2 Acknowledgement

One of us (US) would like to acknowledge the hospitality of the High Energy Physics Group of the University of Hawaii where this work was initiated and one (S.P) would like to thank the Physical Research Laboratory for their hospitality. This work was supported in part by US D.O.E under contract DE-AMO3-76SF-00325.

References

- [1] J.C. Pati and A. Salam, Phys. Rev. **D 10** (1974) 275; R.N. Mohapatra and J.C. Pati, Phys. Rev. **D 11** (1975) 566.
- [2] G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. **48**, 848 (1982); G. Ecker and W. Grimus, Nucl. Phys. **B258**, 328 (1985).
- [3] A. Datta and A. Raychaudhuri, Phys. Lett. **B122**, 392 (1982); P. Basak, A. Datta and A. Raychaudhuri, Z. Physik **C20**, 305 (1983).
- [4] See, e.g., J. Freeman, in ‘Particle Phenomenology in the 90’s’, eds. A. Datta, P. Ghose and A. Raychaudhuri (World Scientific, 1992).

- [5] G. Bhattacharyya, A. Datta, S.N. Ganguli and A. Raychaudhuri, Mod. Phys. Lett. (*Brief Review*) **A6**, (1991) 2557; G. Altarelli *et al.*, Phys. Lett. **B245** (1990) 669; G. Altarelli *et al.*, Nucl. Phys. **B342** (1990) 15; G. Altarelli *et al.*, Mod. Phys. Lett. **A5** (1990) 495; G. Bhattacharyya, A. Datta, S.N. Ganguli and A. Raychaudhuri, Phys. Rev. Lett. **64** (1990) 2870.
- [6] G. Altarelli *et al.*, Phys. Lett. **B245** (1990) 669; G. Altarelli *et al.*, Nucl. Phys. **B342** (1990) 15; G. Bhattacharyya, A. Datta, A. Raychaudhuri and U. Sarkar, Phys. Rev. **D 47** (1993) R3693.
- [7] OPAL Collab., M.Z. Akrawy *et. al.*, Phys. Lett. **B 253** (1991) 511; ALEPH Collab., D. Decamp *et. al.*, CERN preprint CERN-PPE/91-19; DELPHI Collab., P. Abreu *et. al.*, CERN preprint CERN-PPE/90-163; L3 Collab., B. Adeva *et. al.*, L3 preprint 024 (1990).
- [8] H.Georgi, H.R.Quinn and S. Weinberg, Phys. Rev. Lett. **33** 451 (1974); See also A. Masiero in "Grand Unification with and without Supersymmetry and Cosmological Implications", International School for Advanced Studies Lecture Series No. 2, published by World Scientific, (1984), p.1; and A. Zee (ed.) "Unity of Forces in the Universe", Vol. 1, published by World Scientific, 1982.
- [9] U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. **B 260** (1991) 447; U. Amaldi *et. al.*, CERN Preprint No. CERN PPE/91-233.
- [10] P. Langacker and M. Luo, *ibid.* **D 44** (1991) 817; B. Brahmachari, U. Sarkar and K. Sridhar, Phys. Lett. **297 B** (1992) 105;; A. Galli, PSI preprint unpublished
- [11] N.G. Deshpande, E. Keith and P.B. Pal, Phys. Rev. **D 46** (1992) 2261; *ibid.* **D 47** (1993) 2892.

- [12] A. Datta, S. Pakvasa and U. Sarkar, Phys. Lett. **B 313** (1993) 83.
- [13] N.G. Deshpande, E. Keith and T. Rizzo, Phys. Rev. Lett. **70** (1993) 3189.
- [14] D. Chang, R.N. Mohapatra and M.K. Parida, Phys. Rev. Lett. **52** (1984) 1072;
D. Chang, R.N. Mohapatra, J. Gipson, R. E. Marshak and M. K. Parida, Phys. Rev. **D 31** (1985) 1718.
- [15] C.T. Hill, Phys. Lett. **B 135** (1984), 47; C. Wetterich, Phys. Lett. **B 110** (1982), 384; J. Ellis and M.K. Gaillard, Phys. Lett. **B 88** (1979) 315; Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52** (1984) 875.
- [16] M.K. Parida, P.K. Patra and A.K. Mohanty, Phys. Rev. **D 39** (1989) 316; B. Brahmachari, P.K. Patra, U. Sarkar and K. Sridhar, Mod. Phys. Lett. **A 8** (1993) 1487.
- [17] P.G.O. Freund, Nucl. Phys. **B 209** (1982) 146.
- [18] R.N. Mohapatra and M.K. Parida, Phys. Rev. **D 47** (1993) 264.
- [19] D.R.T. Jones, Phys. Rev. **D 25** (1982) 581.
- [20] R.N. Mohapatra and G. Senjanovic, Phys. Rev. **D 27** (1983) 1601; F. Del.Aguila and L. Ibanez, Nucl.Phys. **B 177** (1981) 60.
- [21] S. Weinberg, Phys. Lett. **B 91** (1980) 51; L. Hall, Nucl. Phys. **B 178** (1981) 75.

0.3 Figure Captions

Fig. 1: The allowed regions in ϵ and ϵ' space for D-conserved non-susy SO(10) for pairs of M_I and M_U . For M_I not equal to M_U the upper and the lower regions correspond to case (a) and case (b), the two cases considered for threshold corrections.

Fig. 2: The allowed regions in ϵ and ϵ' space for D-conserved SO(10).

Fig. 3: The allowed regions in ϵ and ϵ' space for D-nonconserved SO(10).

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402356v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402356v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402356v1>